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$$\therefore x_1 = -\frac{(b_1 - a_1)(b_1 - a_2) \dots (b_1 - a_n)}{(b_1 - b_2)(b_1 - b_3) \dots (b_1 - b_n)}, \quad x_r = -\frac{(b_r - a_1)(b_r - a_2) \dots (b_r - a_n)}{(b_r - b_1)(b_r - b_2) \dots (b_r - b_n)}.$$

GEOMETRY.

203. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Show that two parabolae can always be drawn through the vertices of a triangle to touch its circumcircle at a vertex, and that the axes of these pairs of curves are orthogonal. Show that any triangle may be circumscribed by a conic so that the tangents at each vertex are parallel to the opposite side.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

In order that $l^2\gamma + m^2\alpha + n^2\beta = 0$ may be a parabola we must have

$$l^2a^2 + m^2b^2 + n^2c^2 = 2ablm + 2bcmn + 2acln.$$

For tangency at a vertex we must have $l\beta + ma = 0$, also $a\sin B + \beta\sin A = 0$.

$$\therefore l/m = \sin A / \sin B = a/b, \text{ or } l = ma/b.$$

$$\therefore m^2a^2/b^2 + m^2b^2 + n^2c^2 = 2a^2m^2 + 2bcmn + 2a^2cmn/b.$$

$$\therefore m^2(a^2 - b^2)^2 + n^2b^2c^2 = (2b^3c + 2a^2bc)mn.$$

$$\therefore n = \frac{(a \pm b)^2}{bc} m.$$

$$\therefore ac\beta\gamma + bc\gamma a + (a \pm b)^2\alpha\beta = 0 \text{ is the equation to the two parabolae.}$$

If two of the sides be taken as axes, the Cartesian equation is $(x \pm y)^2 + bx + ay = 0$. In order that the tangents at each vertex may be parallel to the opposite sides we must have $l/m = b/a$, $m/n = c/b$.

$$\therefore \beta\gamma/a + \gamma a/b + \alpha\beta/c = 0 \text{ is the conic required, and this conic is an ellipse.}$$

II. Solution by G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let the triangle be formed by the lines a, β, γ . The tangent at the vertex $a\beta$ to the circumcircle is

$$a\beta + ba = 0.$$

The equation of any circumconic having this line for a tangent may be written

$$a\beta\gamma + b\gamma a + \lambda a\beta = 0.$$

The coördinates of the center, *i. e.*, of the pole of the line at infinity, are found to be

$$\lambda ac - a^2 + ab^2, \quad \lambda bc - b^3 + a^2b, \quad \lambda a^2 + \lambda b^2 - \lambda^2 c.$$

If this point lies on the line at infinity, the conic will be a parabola. This condition gives $\lambda = (a \pm b)^2/c$, and hence there are two parabolae satisfying this condition. The coördinates of the centers become

$$2ab(b \pm a), \quad 2ab(a \pm b), \quad \mp 2ab(a \pm b)^2/c.$$

From the first two coördinates we see that the axes of these parabolæ are the bisectors of the angle at $a\beta$.

Problem 204 was also solved by D. B. Northrup, Schenectady, N. Y.

205. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Having given any two systems of conjugate semi-diameters of an ellipsoid, the parallelopiped which has any three for continuous edges is equal to that which has the other three for continuous edges.

Remark by H. B. LEONARD, A.B., Chicago, Ill.

Solved in C. Smith's "An Elementary Treatise on Solid Geometry," page 76.

CALCULUS.

166. Proposed by T. N. HAUN, Mohawk, Tenn.

Find the volume of the solid formed by the revolution of the curve $(y^2 + x^2) = a^2(x^2 - y^2)$ round the axis of x .

II. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

This equation represents singly the two lines $y = \pm x/(a^2 - 1)/(a^2 + 1)$, and the surface generated is simply a right circular cone whose area, volume, bounded by the plane $x = c$, is simply $\frac{1}{3}\pi c^3(a^2 - 1)/(a^2 + 1)$, and where $a \neq 1$.

168. Proposed F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

The tangent of what Cartesian curve makes an x -intercept always m times as long as the corresponding y -intercept?

II. Solution by G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let the tangent at $P(x, y)$ meet the axes in the points A and B . Let the perpendicular from P to OY meet it at M . Call θ the angle APM .

Then $PB = m \cdot PA$, $OM = m \cdot AM$; i. e., $y = m \cdot PM \cdot \tan \theta = mx(dy/dx)$.

$\therefore y^m = cx$.

AVERAGE AND PROBABILITY.

145. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

In each quadrant of a given circle, a circle is described at random. A point is taken at random in each of these circles. What is the average area of the quadrilateral formed by joining with straight lines these four points?